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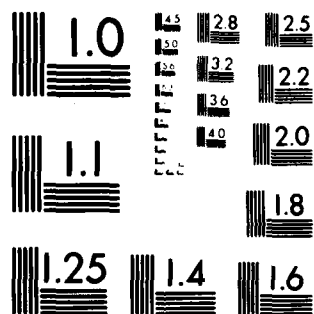   AUG 80   R C LEACHMAN   N00014-76-C-0134
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PRODUCTION PLANNING FOR MULTI-RESOURCE NETWORK SYSTEMS

by

ROBERT C. LEACHMAN

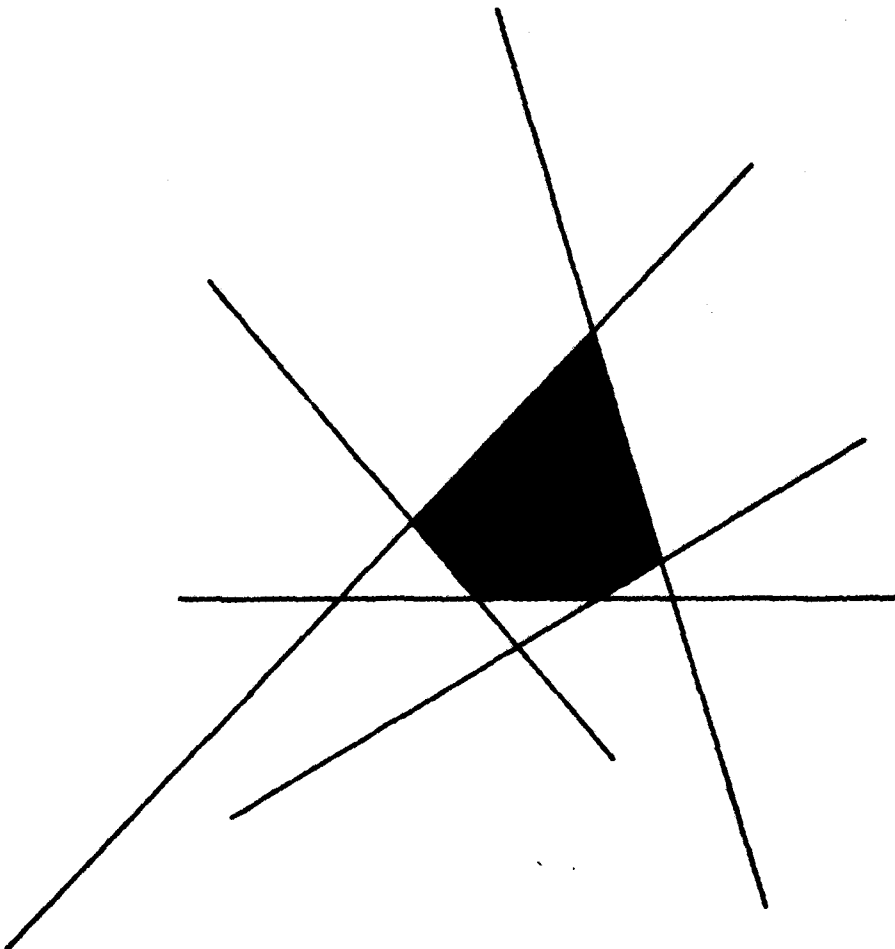# OPERATIONS RESEARCH CENTER

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PRODUCTION PLANNING FOR MULTI-RESOURCE NETWORK SYSTEMS[†]

by

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ABSTRACT

Production planning for large-scale production
systems requiring the allocation of numerous re-
sources is considered. It is demonstrated how the
dynamic activity analysis developed by Shephard
leads to linear programming solutions of produc-
tion planning problems. Three types of planning
problems are formulated: maximization of output
levels for a given time horizon; minimization of
production duration for given output histories;
and minimization of production costs for given out-
put histories.

Production Planning for Multi-Resource Network Systems

Robert C. Leachman
University of California, Berkeley

1. Introduction

Previous efforts (von Lanzenauer [4] and Candea [1]) to mathematically model capacitated, multistage production systems have been motivated by manufacturing shop environments, in which many products are to be produced using a given network of facilities.* The problem considered is to determine workforce levels and product lot sizes in each time period so as to minimize costs to meet external demand schedules [1] [4].

The focus of this paper is planning for production systems in which the production network elements are dedicated to producing a single product, but allocation of numerous resources among the elements is required, and other kinds of production planning problems are posed. Shephard et. al. [7] [8] have developed a continuous flow dynamic activity analysis model of production, in which a network of activities characterizes the component tasks of production. Required facilities and other resources are considered as inputs to be allocated among the activities. The presentation in [8] is taken as an appropriate point of departure here. In the following, this model is extended to include inventory capacities as in [4], initial inventories of intermediate products, and classes of exogenous inputs.

Three types of production planning problems are formulated and solved using linear programming methods. The problem types considered are maximization of output accumulations by a given horizon; minimization of production duration for required output histories; and minimization of costs for given output requirements.

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* No attempt is made here to reference multistage modeling efforts of uncapacitated systems or pure serial or parallel structures; for a survey of such efforts, see Candea [1].

2. The Model

Following Shephard and Al-Ayat [8], the production system is viewed as a network of production activities which are denoted by $A_1, A_2, \ldots, A_N$. In the network, nodes represent activities and arcs indicate *intermediate product transfers*, ie, the use of each activity's output as input by other activities. The operation of each activity A_i is measured in terms of an *intensity function* $z_i(t)$, $t=0,1,2,\cdots$, whose value at time t indicates activity input during $[t,t+1)$ and output at time $(t+1)$ when taken with technical coefficients defined as follows:

(a) $c_i(t)$, $t=0,1,2,\ldots$, $i=1,\ldots,N$, where $c_i(t)$ is the amount of output of activity A_i at time t per unit intensity of activity A_i.

(b) $a_{ik}(t)$, $t=0,1,2,\ldots$, $k=1,2,\ldots,NK$, $i=1,\ldots,N$, where $a_{ik}(t)$ is the amount of exogenous input type k required at time t per unit intensity of activity A_i. The first $NS \leqslant NK$ inputs are designated *non-storable resources* which cannot be accumulated; the remaining exogenous inputs can be accumulated, and are termed *storable resources*.

(c) $\bar{a}_{ij}(t)$, $t=0,1,2,\ldots$, $i=1,\ldots,N$, $j=1,\ldots,N$, where $\bar{a}_{ij}(t)$ is the amount of intermediate product from activity A_i required at time t per unit intensity of operating A_j.

For a time horizon T for production activity, we introduce the following technical limitations on the system:

(a) $\left\{\bar{z}_i(t)\right\}_{i=1,\ldots,N}^{t=0,1,2,\ldots,T-1}$, the activity *intensity bounds*, natural bounds resulting from available workspace and other limitations not considered as exogenous input;

(b) $\left\{X_k(t)\right\}_{k=1,\ldots,NS}^{t=0,1,2,\ldots,T-1}$, the time histories of non-storable resource levels available for input to the system;

(c) $\left\{Y_k(t)\right\}_{k=NS+1,\ldots,NK}^{t=0,1,\ldots,T-1}$, the time histories of storable resources made available to the system, where

$$\sum_{\tau=0}^{t} Y_k(\tau)$$

is the cumulative amount of resource k supplied during $[0,t)$;

(d) $\left\{inv_i^0\right\}_{i=1,\ldots,N}$, the initial inventories of activity product for intermediate uses; and

(e) $\left\{cap_i(t)\right\}_{i=1,\ldots,N}^{t=0,1,\ldots,T-1}$, the bounds on accumulations of activity product awaiting inter-

mediate uses, arising from limited storage capacity for in-process inventories.

For our purposes here, the intensity $z_i(t)$ of activity A_i, $i=1,\ldots,N$, on each time inter-
val $[t,t+1)$, $t=0,1,2,\ldots,T-1$, shall be partitioned into effort producing intermediate product,
$z_i^I(t)$, and effort producing final product, $z_i^F(t)$, where

$$z_i^I(t) + z_i^F(t) = z_i(t). \tag{1}$$

These variables indicate the allocation of activity output produced during $[t,t-1)$ to final and
intermediate uses.

A *production plan* is a specification over some finite period $[0,T)$ of the activity intensities

$$\left\{z_i^I(t),\ z_i^F(t)\right\}_{i=1,\ldots,N}^{t=0,\ldots,T-1}.$$

Such a plan is said to be *feasible for L(T)* if the plan belongs to the set $L(T)$ defined by the fol-
lowing inequalities:

L(T)1. $\sum_{i=1}^{N} a_{ik}(t)\left[z_i^I(t) + z_i^F(t)\right] \leqslant X_k(t)$, $k=1,\ldots,NS$, $t=0,1,\ldots,T-1$.

L(T)2. $\sum_{\tau=0}^{t} \sum_{i=1}^{N} a_{ik}(\tau)\left[z_i^I(\tau) + z_i^F(\tau)\right] \leqslant \sum_{\tau=0}^{t} Y_k(\tau)$, $k=NS+1,\ldots,NK$,

$t=0,\ldots,T-1$.

L(T)3. $\sum_{\tau=1}^{t} \sum_{i=1}^{N} \bar{a}_{ji}(\tau)\left[z_i^I(\tau) + z_i^F(\tau)\right] - \sum_{\tau=0}^{t-1} c_j(\tau+1)z_j^I(\tau) \leqslant inv_j^0$.

$j=1,\ldots,N$, $t=1,\ldots,T-1$, *and*

$$\sum_{i=1}^{N} \bar{a}_{ji}(0)\left[z_i^I(0) + z_i^F(0)\right] \leqslant inv_j^0, \quad j=1,\ldots,N.$$

L(T)4. $\sum_{\tau=0}^{t-1} c_j(\tau+1)z_j^I(\tau) - \sum_{\tau=1}^{t} \sum_{i=1}^{N} \bar{a}_{ji}(\tau)\left[z_i^I(\tau) + z_i^F(\tau)\right] \leqslant cap_j(t) - inv_j^0$.

$j=1,\ldots,N$, $t=1,\ldots,T-1$, *and*

$$- \sum_{i=1}^{N} \bar{a}_{ji}(0) \left[z_i^I(0) + z_i^F(0) \right] \leqslant cap_j(0) - inv_j^0, \quad j=1, \ldots, N.$$

L(T)5. $z_j^I(t) + z_j^F(t) \leqslant \bar{z}_j(t), \quad j=1, \ldots, N, \; t=0, \ldots, T-1,$

$z_j^I(t), \; z_j^F(t) \geqslant 0, \quad j=1, \ldots, N, \; t=0, \ldots, T-1.$

Constraints L(T)1 and L(T)2 express resource limitations. Constraints L(T)3 insure adequate intermediate product transfers occur to support production activity, while constraints L(T)4 insure that inventories of intermediate products do not exceed capacities. Finally, constraints L(T)5 limit intensities to non-negative values less than intensity bounds.

The set of linear inequalities L(T) constitutes a *continuous flow model* of production, in which any positive intensity of activity operation supplies completed product to final or intermediate uses, or to inventory. In the case that intermediate products of a system are large, discrete units, precedence relations occur between activities output unit by output unit, and constraints L(T)3 and L(T)4 must be modified. This case will not be treated here, and the reader is referred to [6], in which a dynamic activity analysis was developed on a critical path analysis network.

In the case more than one activity produces a certain product, constraints L(T)3 and L(T)4 must be modified for the activities in question. See [6] and [8]. However, with such revisions L(T) still constitutes a set of linear inequalities. For simplicity of exposition, we assume in what follows that no two activities produce the same product.

3. Production Programming

3.1. Output Maximization

In this section, programs are formulated for the maximization of value or mix functions of final output accumulations. We consider first the case where a specific product mix of final output is desired, and the problem is to maximize the scale of this mix accumulated by a time horizon T.

Let z_{N+1} be a variable indexing the scale of the accumulation by time T. The amounts of the various products will be related by coefficients

$$\bar{a}_{i,N+1}, \quad i=1, \ldots, N,$$

where $\bar{a}_{i,N+1}z_{N+1}$ is the amount of final product from activity A_i accumulated. The problem in question is formulated as a linear program as follows.

Maximize z_{N+1}

subject to

O1. $\bar{a}_{i,N+1}z_{N+1} \leqslant \sum_{t=0}^{T-1} c_i(t+1)z_i^F(t)$, $i=1,\ldots,N$.

O2. $\left\{ z_i^I(t),\ z_i^F(t) \right\}_{i=1,\ldots,N}^{t=0,\ldots,T-1} \in L(T)$.

O3. $z_{N+1} \geqslant 0$.

In general, the program involves $[5(N) + NK](T) + N + 1$ variables, and $[3(N) + NK](T) + N$ constraints. Clearly, the time horizon (ie, the number of time periods) is the most sensitive factor in terms of problem size which can be handled. The structure of the constraint set can be modified by rewriting constraints L(T)3 and L(T)4 in terms of intermediate product inventory variables

$$\left\{ inv_j^I(t) \right\},\ t=0,\ldots,T-1,$$

which are the slack variables defined by constraints L(T)3. Using these variables, we reformulate constraints L(T)3 and L(T)4 as follows:

L(T)3. $\sum_{i=1}^{N} \bar{a}_{ji}(t)\left[z_i^I(t) + z_i^F(t) \right] - c_i(t)z_i^I(t-1) - inv_j^I(t-1) + inv_j^I(t) = 0$,

$j=1,\ldots,N,\ t=1,\ldots,T-1,$ *and*

$$\sum_{i=1}^{N} \bar{a}_{ji}(0)\left[z_i^I(0) + z_i^F(0) \right] + inv_j^I(0) = inv_j^0,\ j=1,\ldots,N.$$

L(T)4. $inv_j^I(t) \leqslant cap_j(t),\ j=1,\ldots,N,\ t=0,\ldots,T-1.$

With this revision, it is evident that the constraints L(T)1, L(T)4 and L(T)5 apply only time period by time period, and the constraint matrix exhibits partial block diagonal structure. Potential is thus offered for application of large-scale programming procedures such as

decomposition. (See [5].)

We next consider the case where the value of output produced is to be maximized. We suppose each product i has a constant unit price p_i. The maximum value of output accumulated from production activity during $[0, T)$ is then given by the optimum of the following linear program.

$$\text{Maximize} \quad \sum_{t=0}^{T-1} \sum_{i=1}^{N} p_i c_i(t+1) z_i^F(t)$$

subject to

$$\left\{ z_i^I(t), \ z_i^F(t) \right\}_{i=1,\ldots,N}^{t=0,\ldots,T-1} \in L(T).$$

The remarks about problem size and structure concerning the previous program apply here as well, as only the product mix variable and the N constraints O1 have been deleted.

3.2. Time Minimization

In this kind of planning problem, there are final output demands which must be met, but the overall production duration is to be minimized. Final output demands are expressed in cumulative terms as follows. Let

$$\hat{U}_i(t), \ t=1, 2, \ldots, T, \ i=1, \ldots, N,$$

denote the required cumulative delivery of final product i by time t. Here, we are considering the situation where early delivery of final products is acceptable or even desirable. These demands imply constraints

$$\sum_{\tau=0}^{t-1} c_i(\tau+1) z_i^F(\tau) \geq \hat{U}_i(t), \ i=1, \ldots, N, \ t=1, \ldots, T. \tag{2}$$

We first consider the problem of finding the latest starting time for production activity sufficient to satisfy (2). A feasible production plan for this problem would satisfy the linear inequalities (2) and $L(T)$. An optimal plan would have the characteristic that

$$z_i(0) = z_i(1) = z_i(2) = \cdots = z_i(t_0) = 0, \ i=1, \ldots, N, \tag{3}$$

where t_0 is as large as possible. Such a plan may be found (if one exists) by solving a sequence

of Phase I linear programs (see [2]) as follows.

The set of inequalities under consideration is of the form

$$\mathbf{Az} + \mathbf{Bx} = \mathbf{b}$$
$$\mathbf{z} \geqslant 0, \ \mathbf{x} \geqslant 0,$$

(4)

where

$$\mathbf{z} = \left[z_1^I(0), \ z_1^F(0), \ \ldots, z_N^I(0), \ z_N^F(0), \ z_1^I(1), \ z_1^F(1), \ \ldots, z_N^I(1), \ z_N^F(1), \ \ldots, \right.$$

$$\left. z_1^I(T-1), \ z_1^F(T-1), \ \ldots, z_N^I(T-1), \ z_N^F(T-1) \right];$$

$$\mathbf{x} = (x_1, \ \ldots, x_{m+n}) \ ;$$

$$m = (N)(T), \ n = (NK)(T) + 3(N)(T), \ l = 2(N)(T) \ ;$$

A is the $(m+n) \times l$ matrix of coefficients of activity intensities in (2) and $\mathbf{L}(\mathbf{T})$, where the first m rows arise from (2);

B is the $(m+n) \times (m+n)$ matrix of coefficients of slack variables for said constraints; and

b is the $(m+n)$ vector of right hand side constant terms of the constraints.

For the inequalities organized in this fashion, the solution algorithm is presented below:

Step 0. Initialize $\tau = T-1$.

Step 1. Solve the Phase I problem with the first $(N)(\tau)$ columns of the tableau corresponding to (4) deleted. If a feasible solution is found, stop; then

$$\left\{ z_i^I(t), \ z_i^F(t) \right\}_{i=1,\ldots,N}^{t=0,\ldots,T-1}$$

is an optimal production plan. If the problem is infeasible, go to Step 2.

Step 2. If $\tau = 0$, stop; then the set of inequalities is infeasible. Otherwise, decrease τ to $\tau-1$ and go to Step 1.

The algorithm is seen to initially ignore all columns in the tableau corresponding to activity intensities in periods before time $(T-1)$, and to then attempt to find a basic solution. If none can be found, columns corresponding to

$$z_i^I(T-2), \ z_i^F(T-2), \ i=1, \ \ldots, N,$$

are also considered. The algorithm continues to allow the use of columns corresponding to activity operation one time period before the earliest period of activity operations allowed by the previous iteration. The algorithm terminates either the first time a feasible basic solution is found, or else all columns are adjoined without finding one. In the former case, an optimal production plan is found, and in the latter case, the output schedule (2) is infeasible for the limitations $L(T)$.

We next consider the problem of finding the earliest time all product accumulations can be completed. It is immediately apparent that an approach similar to that considered above can be used to solve this problem. A sequence of Phase I procedures is again suggested, but in this case starting with the possibility of positive activity intensities only during $[0, 1)$, and proceeding forwards in time. Later production activity is allowed period by period until either the first time a feasible basic solution is found, or else the horizon is reached without finding one. In the former case, an optimal production plan is obtained, and in the latter case, the output schedule is infeasible for the limitations imposed.

3.3. Cost Minimization

In this section we formulate the problem of determining a minimum cost production plan which meets a given final output schedule expressed in the form of (2).

Non-storable resources are assumed to have *capacity costs* corresponding to the peak demands for each such resource. These resources cannot be accumulated, so that the production system must have the capability to accommodate peak loads. Storable resources, however, have *prices;* these resources account for the variable cost of production activity. It is assumed that storable resources can be procured as required, so that inventories of same are ignored. We assume intermediate product inventories also have capacity costs, corresponding to peak storage requirements. These inventories will also bear holding costs representing opportunity charges for unproductive capital.

Assuming linear capacity costs, the problem is formulated as a linear program as follows. Let

$$C^x(t) = \left[C_1^x(t), \ldots, C_{N_S}^x(t) \right],$$

be the vector of costs per unit capacity for non-storable resources maintained during $[t, t+1)$; let

$$\mathbf{C}^y(t) = \left[C_{NS+1}^y(t), \ldots, C_N^y(t)\right],$$

be the price vector for storable resources procured for use during $[t,t+1)$; let

$$\mathbf{C}^z(t) = \left[C_1^z(t), \ldots, C_N^z(t)\right],$$

be the vector of costs per unit storage capacity maintained during $[t,t+1)$ for intermediate products; and let

$$\mathbf{H}(t) = \left[H_1(t), \ldots, H_N(t)\right],$$

be the holding costs for intermediate products held during $[t,t+1)$.

To serve as variables in the minimization, let

$$\mathbf{X} = \left[X_1, \ldots, X_{NS}\right],$$

denote the peak requirements in any unit time interval of non-storable resources; let

$$\mathbf{cap} = \left[cap_1, \ldots, cap_N\right],$$

denote the required intermediate product inventory storage capacities; and let

$$\left\{inv_i^I(t), \; z_i^I(t), \; z_i^F(t)\right\}_{i=1, \ldots, N}^{t=0, \ldots, T-1}.$$

be the inventory and intensity variables as before.

For given intensity bounds

$$\left\{\bar{z}_i(t)\right\}_{i=1, \ldots, N}^{t=0, \ldots, T-1}$$

and initial intermediate product inventories

$$\left\{inv_i^0\right\}_{i=1, \ldots, N},$$

the minimum cost production plan meeting the output schedule (2) is given by the optimum of the linear program

$$\text{Minimize} \quad \sum_{t=0}^{T-1} \mathbf{C}^x(t) \cdot \mathbf{X} + \sum_{t=0}^{T-1} \sum_{k=NS+1}^{VK} \sum_{i=1}^{V} C_k^x(t) a_{ik}(t) \left[z_i^I(t) + z_i^F(t) \right]$$

$$+ \sum_{t=0}^{T-1} \mathbf{C}^z(t) \cdot \mathbf{cap} + \sum_{t=0}^{T-1} \sum_{i=1}^{V} H_i(t) \, inv_i^I(t)$$

subject to

C1. $\displaystyle\sum_{\tau=0}^{t-1} c_i(\tau+1) z_i^F(\tau) \geqslant \hat{U}_i(t)$, $i=1,\ldots,N$, $t=1,\ldots,T$.

C2. $\displaystyle\sum_{i=1}^{V} a_{ik}(t)\left[z_i^I(t) + z_i^F(t) \right] - X_k \leqslant 0$, $k=1,\ldots,NS$, $t=0,1,\ldots,T-1$.

C3. $\displaystyle\sum_{i=1}^{V} \bar{a}_{ji}(t)\left[z_i^I(t) + z_i^F(t) \right] - c_j(t) z_j^I(t-1) - inv_j^I(t-1) + inv_j^I(t) = 0$,

$j=1,\ldots,N$, $t=1,\ldots,T-1$, and

$\displaystyle\sum_{i=1}^{V} \bar{a}_{ji}(0)\left[z_i^I(0) + z_i^F(0) \right] + inv_j^I(0) = inv_j^0$, $j=1,\ldots,N$.

C4. $inv_j^I(t) - cap_j(t) \leqslant 0$, $j=1,\ldots,N$, $t=0,\ldots,T-1$.

C5. $z_j^I(t) + z_j^F(t) \leqslant \bar{z}_j(t)$, $j=1,\ldots,N$, $t=0,\ldots,T-1$.

C6. $\mathbf{X} = \left[X_1,\ldots,X_{NS} \right] \geqslant 0$,

$\mathbf{cap} = \left[cap_1,\ldots,cap_N \right] \geqslant 0$,

$\mathbf{inv}^I(t) = \left[inv_1^I(t),\ldots,\bar{inv}_N^I(t) \right] \geqslant 0$, $t=0,\ldots,T-1$,

$z_j^I(t)$, $z_j^F(t) \geqslant 0$, $j=1,\ldots,N$, $t=0,\ldots,T-1$.

Here constraints C2 define the required non-storable resource capacities, and constraints C4 define the required intermediate product storage capacities. Constraints C3 and C5 deal with inventory balance and intensity bounds in the same manner as the treatment of output maximization problems, while constraints C1 repeat (2).

In general, the program includes $(4N + NS)(T+1)$ variables and $(4N + NS)(T)$ constraints. As before, the fineness of the time grid is the most sensitive factor in terms of the problem size which can in practice be solved. A bordered angular configuration for the

constraint matrix is now displayed, in which constraints C2, C4, and C5 exhibit a block diagonal structure with coupling variables. Although this is a more difficult structure than that for the output maximization problems, nonetheless it can be exploited. See [5].

As an alternative to the constant capacities for each non-storable resource defined by constraints C2, one may allow capacities to be adjusted from time period to time period according to linear costs. Many authors have formulated labor workforce levels in this fashion, allowing hiring and firing in each period. See for example [3] or [4]. Such formulations may be integrated here as appropriate.

Acknowledgements

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